Exactly Solvable Ginzburg-Landau theories of Superconducting Order Parameters coupled to Elastic Modes

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We consider two families of exactly solvable models describing thermal fluctuations in two-dimensional superconductors coupled to phonons living in an insulating layer, and study the stability of the superconducting state with respect to vortices. The two families are characterized by one or two superconducting planes. The results suggest that the effective critical temperature increases with the thickness of the insulating layer. Also the presence of the additional superconducting layer has the same effect.

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I. INTRODUCTION

Recently Fateev [1,2] introduced two families of integrable models which can be interpreted as Ginzburg-Landau free energy functionals describing thermal (classical) fluctuations in superconducting films. According to this interpretation the models of the first family, which we shall further call type I models, describe a single superconducting layer deposited on an insulating substrate consisting of n layers. The superconducting order parameter interacts with the elastic modes of the substrate. The effect of the interaction is to shift the local transition temperature. Type II models describe the situation of a double layer where an insulator is sandwiched between two superconducting films. The latter ones may have different critical temperatures.

These models provide a rare opportunity to go beyond weak coupling description and obtain non-perturbative results for layered superconductors interacting with an insulating stratum. In the case of type-I models our principal interest is to find how the Kosterlitz-Thouless transition temperature depends on the thickness of the substrate. Type-II models allow us to study the effects of interaction between the superconducting order parameters.

The paper is organized as follows. In the next Section we describe the Fateev's models and explain why they can be interpreted as effective Ginzburg-Landau theories for layered superconductors. In Section III we study the stability of the type I models with respect to the vortices. We do a similar analysis for the type II models in Section IV and discuss the results in Section V.

II. EFFECTIVE GINZBURG-LANDAU THEORY

In this section we describe the Fateev's models and construct the effective Ginzburg-Landau free energy. The models of the type I describe a complex bosonic field, Δ, interacting with a n-component real scalar field $\phi = (\phi_1, ..., \phi_n)$. There are different models with slightly different actions for the scalar fields; for our purposes it is sufficient to describe only one of them where the classical (Euclidean) action has the following form:

$$S_n^{(I)} = \frac{1}{\gamma^2}\int \mathrm{d}^2x \left\{ 2\frac{\partial_\mu\Delta\partial_\mu\bar{\Delta}}{1+\Delta\bar{\Delta}} + 2M_0^2\bar{\Delta}\Delta \right. \\ \exp(-\phi_1) + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{M_0^2}{2}\left[2\exp(-\phi_1) + \right. \\ \left. \left. 2\sum_{i=1}^{n-1}\exp(\phi_i - \phi_i + 1) + 2\exp(\phi_n)\right]\right\} \tag{1}$$

This action corresponds to a complex sinh-Gordon model coupled to affine Toda chains. The complete list of the models of the two families is given in [1]. For small n form (1) requires certain modifications, in particular for $n = 0$ it becomes [3]:

$$S_0^{(I)} = \frac{2}{\gamma^2}\int \mathrm{d}^2x \left[\frac{\partial_\mu\Delta\partial_\mu\bar{\Delta}}{1+\Delta\bar{\Delta}} + M_0^2\bar{\Delta}\Delta\right] \tag{2}$$

The type II models describe two complex bosonic fields, $\Delta_{1,2}$, interacting with n elastic modes. Again we present only one specific model where the elastic part coincides with the one described by Eq.(1). The action is the following:

$$S_n^{(II)} = \frac{1}{\gamma^2}\int \mathrm{d}^2x \left[2\sum_{s=1,2}\frac{\partial_\mu\Delta_s\partial_\mu\bar{\Delta}_s}{1+\Delta_s\bar{\Delta}_s} + 2M_0^2\bar{\Delta}_1\Delta_1 \right. \\ \exp(-\phi_1) + 2M_0^2\bar{\Delta}_2\Delta_2\exp(-\phi_n) + \frac{1}{2}(\partial_\mu\phi)^2 \\ \left. -\frac{M_0^2}{2}\left\{2\exp(-\phi_1) + 2\sum_{i=1}^{n-1}\exp(\phi_i - \phi_{i+1}) + 2\exp(\phi_n)\right\}\right] \tag{3}$$

For n=0 the above expression has to be modified as:

$$S_0^{(II)} = \frac{2}{\gamma^2}\int \mathrm{d}^2x \left[\frac{1}{2}\sum_{s=1,2}\frac{\partial_\mu\Delta_s\partial_\mu\bar{\Delta}_s}{1+\Delta_s\bar{\Delta}_s} \right. \\ \left. + \ M_0^2\left\{\bar{\Delta}_1\Delta_1 + \bar{\Delta}_2\Delta_2 + 2(\bar{\Delta}_1\Delta_1)(\bar{\Delta}_2\Delta_2)\right\}\right] \tag{4}$$

As a consequence of the $U(1)$ (type I models) or $U(1)\times U(1)$ (type II models) symmetry, the models have the following conserved charges (here we work in Minkovsky space-time):

$$Q_s = -\frac{2\mathrm{i}}{\gamma^2}\int \mathrm{d}x \frac{(\bar{\Delta}_s\partial_0\Delta_s - \Delta_s\partial_0\bar{\Delta}_s)}{[1+\Delta_s\bar{\Delta}_s]} \tag{5}$$

where $s=1$ for type I and $s=1,2$ for type II models. In presence of external chemical potentials h_s the Hamiltonian is modified: $H = H_0 - \sum_s h_s Q_s$. One can introduce new variables

$$\Delta_s \to \Delta_s \mathrm{e}^{\mathrm{i}th_s} \tag{6}$$

to remove the terms with one time derivative. Then we obtain the general form of the Ginzburg-Landau free energy for our layered superconductors:

$$F_n^{(\alpha)}/T = \frac{1}{\gamma^2}\int \mathrm{d}^2x \mathcal{S}_n^{(\alpha)} \tag{7}$$

where:

$$\begin{aligned}\mathcal{S}_n^{(I)} &= 2\frac{\partial_\mu\Delta\partial_\mu\bar{\Delta} - h^2\Delta\bar{\Delta}}{1+\Delta\bar{\Delta}} + 2M_0^2\bar{\Delta}\Delta\mathrm{e}^{-2\phi_1} \\ &+ \frac{1}{2}(\partial_\mu\phi)^2 + 2M_0^2\left[\mathrm{e}^{-2\phi_1} + \sum_{i=1}^{n-1}\mathrm{e}^{2(\phi_i-\phi_{i+1})} + \mathrm{e}^{2\phi_i}\right]\end{aligned} \tag{8}$$

and

$$\begin{aligned}\mathcal{S}_n^{(II)} &= \sum_{s=1,2} 2\frac{\partial_\mu\Delta_s\partial_\mu\bar{\Delta}_s - h_s^2\Delta_s\bar{\Delta}_s}{1+\Delta_s\bar{\Delta}_s} + \frac{1}{2}(\partial_\mu\phi)^2 \\ &+2M_0^2\bar{\Delta}_1\Delta_1\exp(-\phi_1) + 2M_0^2\bar{\Delta}_2\Delta_2\exp(-\phi_n) \\ &-\frac{M_0^2}{2}\left[2\exp(-\phi_1) + 2\sum_{i=1}^{n-1}\exp(\phi_i-\phi_{i+1}) + 2\exp(\phi_n)\right]\end{aligned} \tag{9}$$

Clearly the fields Δ_s can be interpreted as a superconducting order parameters coupled to the optical phonon modes ϕ_n. We note that the elastic modes are not harmonic, which is necessary to make the model integrable, but in the limit $\gamma\to 0$ we reproduce the conventional electron-phonon interaction. Indeed, rescaling the phonon field $\phi\to\gamma\phi$, we see that the limit $\gamma\to 0$ corresponds to harmonic phonons. The fact that the spectrum of the Toda chain coincides with the spectrum of harmonic phonons [4] gives us grounds to believe that the unharmonicity present in the model does not influence the qualitative validity of our results. At the same time it is interesting to have a model with phononic spectrum that is not restricted to the harmonic one.

In order to get a better understanding of the nature of the superconducting state, let us have a deeper look on the free energy for the case $n=0$. For the type I models the free energy can be written as follows:

$$F_0^{(I)}/T = \frac{2}{\gamma^2}\int \mathrm{d}^2x\left[\frac{\partial_\mu\Delta\partial_\mu\bar{\Delta}}{1+\Delta\bar{\Delta}} + V(\bar{\Delta},\Delta)\right] \tag{10}$$

with

$$V(\bar{\Delta},\Delta) = M_0^2\bar{\Delta}\Delta - h^2\frac{\bar{\Delta}\Delta}{1+\Delta\bar{\Delta}} \tag{11}$$

For $M_0 < h$ the effective potential for the order parameter has a minimum at $|\Delta|\neq 0$ which signal the appearance of superconductivity. Indeed, expanding the action around the minimum we notice that the quantities $\tau = (M_0^2 - h^2)/\gamma^2$ may be interpreted as the distance from the mean field transition $(T/T_c - 1)$. Increasing h one can go from the disordered to the superconducting state. One can see it more explicitly using a semiclassical analysis valid for $\gamma \gg 1$. Under the transformation

$$\Delta = \sinh\rho e^{i\varphi} \tag{12}$$

the free energy becomes:

$$F_0^{(I)} = \frac{2}{\gamma^2}\int d^2x\left\{(\partial_\mu\rho)^2 + \tanh^2\rho(\partial_\mu\varphi)^2 + V_{eff}^1(\rho)\right\} \tag{13}$$

where:

$$V_{eff}(\rho,h) = M_0^2\sinh^2\rho - h^2\tanh^2\rho \tag{14}$$

The measure of integration in the path integral changes, but this is not important for what we want to discuss here. For $h/2 > M_0$, $V_{eff}(\rho,h)$ has a minimum at $\rho=\rho_0$, where $\exp 2\rho_0 = \sqrt{2}(h/M)$. Thus when h exceeds the threshold a gapless Goldstone mode appears. To see it explicitly we expand the free energy around the minimum. Rewriting $\rho = \rho_0 + \xi$ and keeping only the quadratic terms in ξ, we get:

$$F_0^{(I)} \sim \frac{2}{\gamma^2}\int d^2x\{(\partial_\mu\xi)^2 + (M_0h/2)\xi^2 + \tanh^2\rho_0(\partial_\mu\varphi)^2\}. \tag{15}$$

We can then identify the gapless mode φ, which velocity is equal to the bare one, and a massive field ξ.

A similar analysis can be performed on the type-II models (again we consider only the simplest case $n=0$). In this case the transformation

$$\Delta_s = \sinh\rho_s e^{i\varphi_s} \tag{16}$$

leads to:

$$\begin{aligned}F_0^{(II)} &= \frac{2}{\gamma^2}\int d^2x\{(\partial_\mu\rho_1)^2 + (\partial_\mu\rho_2)^2 \\ &+ \tanh^2\rho_1(\partial_\mu\varphi_1)^2 + \tanh^2\rho_2(\partial_\mu\varphi_2)^2 + V_{eff}^2(\rho_1,\rho_2)\}\end{aligned} \tag{17}$$

where:

$$V_{eff}^2(\rho_1,\rho_2) = \sum_s V_{eff}(\rho_s,h_s) + 2M_0^2\sinh^2\rho_1\sinh^2\rho_2 \tag{18}$$

This potential can develop one or two minima, depending on the values of h_s, thus generating either one or two gapless modes.

From the exact solution one can see that massless modes appear when h exceeds certain threshold value M, where M is a function of the coupling constant γ and parameter M_0 (a similar condition is obtained for the type II models). In view of the above considerations these modes can be interpreted as fluctuations of the superconducting phase and naively one may identify $h = M$ with the onset of superconductivity.

This interpretation is incorrect however, since it does not take into account vortices. Massless phases of the Fateev's models, like any other two-dimensional critical theories with U(1), or $U(1)\times U(1)$, symmetry, may be unstable with respect to vortices. The reason being that the naive approach doesn't take into account non-analytic configurations of the fields, that on a lattice may give a finite contribution to the free energy. The real transition is of the Beresinskii-Kosterlitz-Thouless (BKT) type [5] and occurs at temperature below the mean field transition temperature established by the condition $h = M$. One simple way to see the origin of the vortex configurations of the order parameter field is the following. Transformations (12), (16) violate an important property of the original model (7), namely the periodicity of its action. The order parameter fields of the original models (8), (9) are periodic in φ_s, while this periodicity is lost after the transformations (12), (16). To recover the original periodicity one should add to the forms (13),(17) exponents of the dual field (see for instance [6]). These terms are not contained in the models we consider, therefore the latter ones can provide an adequate description of the superconducting phase only below the BKT transition temperature, where vortices are irrelevant.

In the next sections we will use the exact solution to study the relevance of vortices in our models. To do this we shall need to study more carefully the gapless state and extract from the Bethe Ansatz equations the scaling dimensions of the vortex operators.

III. THE BEREZINSKII-KOSTERLITZ-THOULESS TRANSITION IN THE MONOLAYER MODELS

Let us consider the type-I models first. The Bethe ansatz solution deals with the Minkovsky version of the models. In our analysis we shall benefit from the fact that the ground state energy of the (1 + 1)-dimensional field theory with coupling constant γ is equal to the free energy of the 2-dimensional classical theory with temperature $T = \gamma^2$:

$$F/T = \mathcal{E} \tag{19}$$

In the Minkovsky version the appearance of the gapless state is related to the creation of a condensate of kinks. Then using the Bethe ansatz solution one can express the ground state energy per unit area in terms of solution of the integral equation:

$$\epsilon(\theta) - \int_{-B}^{B} \mathrm{d}\theta' R(\theta - \theta')\epsilon(\theta') = M\cosh\theta - h \tag{20}$$

$$F/T = \mathcal{E} = \frac{M}{2\pi}\int_{-B}^{B} \mathrm{d}\theta \cosh\theta\epsilon(\theta) \tag{21}$$

where $\epsilon(\theta) < 0$ for $|\theta| < B$ and the integration limit B is defined by the condition

$$\epsilon(\pm B) = 0. \tag{22}$$

The kernel R is related to the two-body S-matrix:

$$R(\theta) = \frac{1}{2\pi\mathrm{i}}\frac{\mathrm{d}\ln S(\theta)}{\mathrm{d}\theta} \tag{23}$$

According to [1] its Fourier transform, $R(\omega)$, has the following form:

$$1 - R(\omega) = \frac{\sinh[\pi\omega(1-g)/\alpha]\cosh[\pi\omega(\alpha+2g)/2h]}{\cosh(\pi\omega/2)\sinh(\pi\omega/\alpha)} \tag{24}$$

where

$$g = \frac{4\pi}{\gamma^2 + 4\pi} \quad \alpha = \frac{H\gamma^2 + 4\pi G}{4\pi + \gamma^2} \tag{25}$$

with G, H depending on the model. For model (1) one has $G = H = 2(n+2)$; G, H always scale with n when $n \to \infty$. It was shown by Fateev that

$$\mathcal{E} = -\frac{h^2}{2\pi[1 - R(\omega = 0)]}g(h/M) \tag{26}$$

where $g(\infty) = 1$.

Another quantity that will be useful in the following is the dressed charge $\zeta(\theta)$ [12,13] defined by the following integral equation:

$$\zeta(\theta) - \int_{-B}^{B} \mathrm{d}\theta' R(\theta - \theta')\zeta(\theta') = 1. \tag{27}$$

The kernel R is defined in (24) and the limit B is determined by condition (22).

As we have said, two-dimensional U(1)-symmetric critical points can be unstable with respect to the presence of vortex configurations of the order parameter field. The vortices constitute potentially relevant perturbations which appear in the effective action as exponents of the dual field (see, [5] and, for example , [6]). This operators become irrelevant if their scaling dimension, d_Θ, is greater than two. When a critical theory includes just

one U(1) field α, the scaling dimension of the order parameter $\Delta = \exp(i\alpha)$, d_Δ, is related to the scaling dimension of the vortex perturbation as follows:

$$d_\Delta d_\Theta = 1/4 \tag{28}$$

Hence the superconducting regime exists at

$$d_\Delta < 1/8. \tag{29}$$

The scaling dimension for the primary field Φ is defined as:

$$d_\Phi = \Delta^+ + \Delta^- \tag{30}$$

where the conformal dimensions $\Delta^\pm$ determine the asimptotics of the correlation functions of primary fields:

$$< \Phi_{\Delta^\pm}(x,t)\Phi_{\Delta^\pm}(0,0) >= \frac{\exp(-2i\mathcal{D}p_F x)}{(x-ivt)^{2\Delta^+}(x+ivt)^{2\Delta^-}}. \tag{31}$$

Here $2\mathcal{D}$ is the momentum of the state in units of the Fermi momentum p_F.

In (1+1)-dimensional critical theories the scaling dimension of primary fields are related to the finite size corrections to the ground state energy [8,9]. For models which, as the model in question, has central charge c=1, conformal dimensions are given by:

$$2\Delta^\pm(\Delta N, D, N^\pm) = 2N^\pm + \left(\frac{\Delta N}{2\mathcal{Z}} \pm \mathcal{Z}D\right)^2. \tag{32}$$

The quantum number ΔN is characteristic of the local field under consideration; in the context of this model it represent the number of particles produced by the primary field in consideration. The quantum numbers D and $N^\pm$, generate the tower of excited states, and represent respectively the number of particles that undergo back scattering processes from one Fermi boundary to the other and the number of particles added at $B(N^+)$ or $-B(N^-)$. While ΔN and D are fixed by the local field, $N^\pm$ must be chosen to give the leading asimptotics in the correlation functions, which is equivalent to minimize $\Delta^\pm$. In integrable models the quantity $\mathcal{Z}$ is related to the dressed charge introduced above in the following way:

$$\mathcal{Z} = \zeta(B). \tag{33}$$

For the order parameter operator we have (see Appendix):

$$d_\Delta = \Delta^+(1,0,0) + \Delta^-(1,0,0) = 1/[2\mathcal{Z}]^2. \tag{34}$$

In order to study the stability of our models with respect to vortices we then have to calculate the value of the dressed charge at the Fermi point. We can make two preliminary observations. First, at $B \to \infty$, corresponding to $h/M \to \infty$, we can approximate Eq. (27) by the Wiener-Hopf (WH) one. Then we have:

$$d_\Delta = \frac{1}{4}[1 - R(\omega = 0)] = \frac{1}{4}\frac{\gamma^2}{\gamma^2 + 4\pi} \tag{35}$$

This gives us the upper estimate for existence of the superconductivity:

$$\gamma^2 < 4\pi, \quad g > 1/2 \tag{36}$$

This result holds for *all* type I models. Second, since $R(\theta)$ is a non-singular kernel, at small B we have $\zeta(B) \to 1$ and the scaling dimension is too large for the superconductivity to occur. Therefore there is a line in the $\gamma - h$-plane separating the superconducting and the disordered regions.

As we noted before for model (8) G and H scale like n for large n. Then the kernel (24) becomes a delta function in the limit $n \to \infty$. In this case one has to be a bit careful with determining $\mathcal{Z}$, but the outcome is simple: the scaling dimension is h-independent and is equal to

$$d_\Delta = (1-g)/4 \tag{37}$$

We can study in more details the behavior of Eq.(27) in the regions $B \gg 1$ and $B \ll 1$ and obtain the asymptotics of the line separating the superconducting from the disordered region. The asymptotic form of the scaling dimension at large B can be found from the generalized Wiener-Hopf method [10] and is equal to

$$\begin{aligned} d_\Delta &= \frac{1}{4}(1-g)[1 + a(M/2h)^\mu + ...] \\ \mu &= \frac{2h}{h+2g} \end{aligned} \tag{38}$$

At $g - 1/2 << 1$ the condition $d_\Delta < 1/8$ gives us

$$(h/2M) > f(N)(g - 1/2)^{-1/\mu} \tag{39}$$

with μ approaching 2 at $N \to \infty$. At $n = 0$ we have $\mu = 2/(1+g)$.

To calculate $\mathcal{Z}$ for $B \ll 1$ it is better to use the following identity:

$$2\mathcal{Z} = \pi v_F \frac{\partial D_\epsilon}{\partial h} \tag{40}$$

where D_ϵ is:

$$D_\epsilon = \int_{-B}^{B} d\theta \sigma(\theta), \tag{41}$$

v_F is the Fermi velocity and $\sigma(\theta)$ is the ground state density for $\epsilon(\theta)$. For small B all objects in equation (40) can be calculated explicitly and one has

$$d_\Delta = \frac{1-g}{8}\left(\frac{M}{h-M}\right)^{\frac{1}{2}} \tag{42}$$

We have solved Eq.(27) numerically for various values of n and constructed the phase diagram of the model using Eq.(29) to identify the superconducting region. The results are presented in Fig.1. It is clear that the insulating substrate strongly affects the superconducting transition, in particular with increasing n the distance between the BKT transition temperature and the mean field transition temperature decreases. This means that the effective critical temperature increases with n. To our knowledge this is the first model that displays such characteristic behavior.

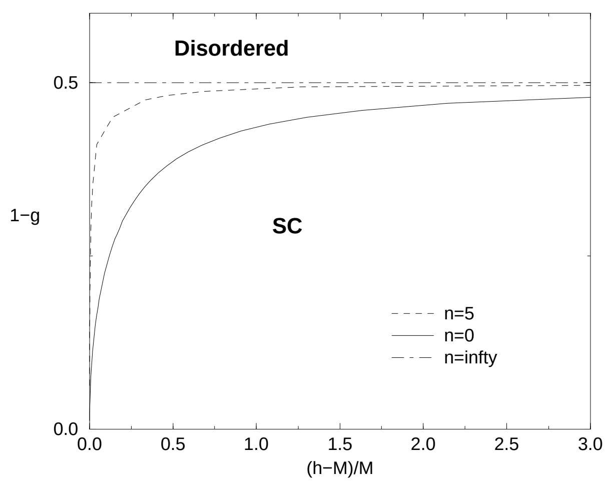

FIG. 1. Phase diagram of the type I models. The curves separate the superconducting region from the disordered one.

IV. EFFECT OF THE INTERACTION BETWEEN SUPERCONDUCTING ORDER PARAMETERS: THE TWO-LAYER MODEL

The situation is considerably more complicated for the case of two-layer models. Since we want to focus our attention on the effects of the interaction between the order parameters we consider only the $n = 0$ version of the model. We have noticed elsewhere that in this case the two gapless modes have the same velocity [11], and hence at this point the model is conformally invariant with the central charge c=2. This result is not general for the Fateev's models but valid only for the specific case $n = 0$. However, the results presented below can be generalized for a case when the two velocities are different. In the latter case the low energy sector is split into two quasi-independent sub-sectors. The system as a whole is not conformally invariant, but each sub-sector is (with its own velocity). Conformal symmetry of each sub-sector makes it possible to generalize the basic relations between the critical exponents and finite size scaling amplitude [15,14] that we will use in the further discussion.

The low-lying excitations for the type-II model with $n = 0$ are described by the following equations:

$$\epsilon_1^{(+)} + \{1 - R_1(\omega)\} * \epsilon_1^{(-)} = \epsilon_1^0(\theta) + \{K_1(\omega)\} * [\epsilon_\nu^{(-)} + \epsilon_{-\nu}^{(-)}] \tag{43}$$

$$\epsilon_{\pm\nu}^{(+)} + \{1 - R_\nu(\omega)\} * \epsilon_{\pm\nu}^{(-)} = \epsilon_{\pm\nu}^0 + \{K_1(\omega)\} * \epsilon_1^{(-)} \tag{44}$$

where we have used the shorthand notation for convolution:

$$\{g(\omega)\} * f = g * f(\theta) = \int_{-\infty}^{+\infty} d\theta' g(\theta - \theta') f(\theta') \tag{45}$$

As usual, $\epsilon_1^{(+)}(\theta) = \epsilon_1(\theta)$ for $|\theta| > B$ and zero otherwise, while $\epsilon_1^{(-)}(\theta) = \epsilon_1(\theta)$ for $|\theta| < B$, and similarly $\epsilon_{\pm\nu}^{(+)}(\lambda) = \epsilon_{\pm\nu}(\lambda)$ for $|\lambda| > Q$ and $\epsilon_{\pm\nu}^{(-)}(\lambda) = \epsilon_{\pm\nu}(\lambda)$ for $|\lambda| < Q$. This relations also fix B and Q. The bare energies are:

$$\epsilon_1^0(\theta) = M\cosh(\theta) - \frac{1}{2}(h_+ + h_-), \quad \epsilon_{\pm\nu}^0 = h_\pm \tag{46}$$

and the quantities $h_\pm$ are related to the chemical potentials:

$$h_\pm = h_1 \pm h_2 \tag{47}$$

Here h_1 and h_2 must be chosen such that $h_\pm$ are always positive. The Fourier transforms of the kernels have the following form:

$$K_1(\Omega) = \frac{\sinh\Omega}{\sinh(\nu\Omega)}, \quad R_\nu(\Omega) = \frac{\sinh((\nu-2)\Omega)}{\sinh(\nu\Omega)} \tag{48}$$

$$R_1(\Omega) = 1 - \frac{\sinh\Omega \sinh\Omega g}{\sinh\frac{(2-g)}{1-g}\Omega \sinh\Omega\nu}, \quad \Omega = \frac{\pi\omega}{2\Lambda}(1-g)$$

and the remaining constants are: $\nu^{-1} = 1-g$, $\Lambda = 2-g$.

For $\frac{1}{2}(h_+ + h_-) > M$ the mode ϵ_1 becomes gapless and, depending on the relative values of h_+ and h_-, it can also induce a second gapless mode. In the following we will consider $h_+ \gg M$. In this case ϵ_ν is always positive and decouples from the other equations such that they become:

$$\epsilon_1(\theta) = \epsilon_1^0(\theta) + \int_{-B}^{B} d\theta' R_1(\theta - \theta')\epsilon_1(\theta') + \int_{-Q}^{Q} d\lambda K_1(\theta - \lambda)\epsilon_{-\nu}(\lambda) \tag{49}$$

$$\epsilon_{-\nu}(\lambda) = \epsilon_{-\nu}^0 + \int_{-Q}^{Q} d\lambda' R_\nu(\lambda - \lambda')\epsilon_{-\nu}(\lambda') + \int_{-B}^{B} d\theta K_1(\lambda - \theta)\epsilon_1(\theta) \tag{50}$$

For $h_- \ll h_+$ it is convenient to invert the kernels in Eq.(50):

$$\left\{\frac{\sinh \nu\Omega}{2\sinh\Omega\cosh[(\nu-1)\Omega]}\right\} * \epsilon_{-\nu}^{(+)} + \epsilon_{-\nu}^{(-)} = \nu h_-/2 + \left\{\frac{1}{2\cosh[(\nu-1)\Omega]}\right\} * \epsilon_1^{(-)}. \quad (51)$$

Using this form in Eq.(49) we get:

$$\epsilon_1^{(+)} + K * \epsilon_1^{(-)} = M\cosh\theta - h_-/2 - \left\{\frac{1}{2\cosh[(\nu-1)\Omega]}\right\} * \epsilon_{-\nu}^{(+)} \quad (52)$$

where

$$K(\Omega) = \frac{\sinh\Omega\cosh[(3\nu-1)\Omega]}{2\sinh\nu\Omega\cosh[(\nu-1)\Omega]\cosh[(\nu+1)\Omega]} \quad (53)$$

Since $\epsilon_{-\nu}^{(+)}$ is very small, to first approximation this reduces to:

$$\epsilon_1(\theta) = \epsilon_1^0(\theta) + \int_{-B}^{B} d\theta' R(\theta-\theta')\epsilon_1(\theta') \quad (54)$$

where the new kernel $R(\Omega)$ is given by:

$$R(\Omega) = 1 - K(\Omega) \quad (55)$$

Clearly $\epsilon_{-\nu}$ become gapless for $h_- < h_c$ defined as:

$$h_c = -\frac{1}{2\nu^2}\int_{-B}^{B} d\theta \frac{\sin\frac{\pi}{\nu}}{\cosh\frac{\pi\theta}{\nu} + \cos\frac{\pi}{\nu}}\epsilon_1(\theta) \quad (56)$$

Already at this stage some interesting features emerge. Namely, in order to get the phase transition on the mean field level, one does not need both fields h_1, h_2 to exceed the critical value for a single superconductor. Instead of the condition $|h_1| \sim |h_2| \sim M/2$ we get a weaker condition $h_1 > M/2$ and $h_2 > h_1 - h_c$.

Now let us discuss how the mean field picture is modified by the vortices. For $h_- > h_c$ when there is only one gapless mode we can calculate the scaling dimension of the order parameter using the procedure of the previous section. However, for $h_- < h_c$ when there are two gapless modes this procedure should be modified. In this case one needs to introduce the dressed charge matrix [14–16],

$$Z = \begin{pmatrix} \mathcal{Z}_{11} & \mathcal{Z}_{1\nu} \\ \mathcal{Z}_{\nu 1} & \mathcal{Z}_{\nu\nu} \end{pmatrix} = \begin{pmatrix} \zeta_{11}(B) & \zeta_{1\nu}(Q) \\ \zeta_{\nu 1}(B) & \zeta_{\nu\nu}(Q) \end{pmatrix}, \quad (57)$$

(we follow the convention introduced in ref. [14,16]) and the function $\zeta(\theta)$ is determined by the following equations:

$$\zeta_{11}(\theta) = 1 + \int_{-B}^{B} \mathrm{d}\theta' R_1(\theta-\theta')\zeta_{11}(\theta') + \int_{-Q}^{Q} \mathrm{d}\lambda K_1(\theta-\lambda)\zeta_{1\nu}(\lambda) \quad (58)$$

$$\zeta_{1\nu}(\lambda) = \int_{-Q}^{Q} \mathrm{d}\lambda' R_\nu(\lambda-\lambda')\zeta_{1\nu}(\lambda') + \int_{-B}^{B} \mathrm{d}\theta K_1(\lambda-\theta)\zeta_{11}(\theta) \quad (59)$$

$$\zeta_{\nu 1}(\theta) = \int_{-Q}^{Q} \mathrm{d}\lambda K_1(\theta-\lambda)\zeta_{\nu\nu}(\lambda) + \int_{-B}^{B} \mathrm{d}\theta R_1(\theta-\theta')\zeta_{\nu 1}(\theta') \quad (60)$$

$$\zeta_{\nu\nu}(\lambda) = 1 + \int_{-B}^{B} \mathrm{d}\theta K_1(\lambda-\theta)\zeta_{\nu 1}(\theta) + \int_{-Q}^{Q} \mathrm{d}\lambda R_\nu(\lambda'-\lambda)\zeta_{\nu\nu}(\lambda') \quad (61)$$

We note that the structure of this equations is very similar to the one for the Hubbard model away from half filling, where the two gapless modes correspond to spin and charge excitations [15,16].

In this general case the formula for the conformal dimensions are given by:

$$2\Delta_1^{\pm}(\mathbf{\Delta N}, \mathbf{D}, \mathbf{N}^{\pm}) = 2N_1^{\pm} + \left(\mathcal{Z}_{11}D_1 + \mathcal{Z}_{\nu 1}D_\nu \pm \frac{\mathcal{Z}_{\nu\nu}\Delta N_1 - \mathcal{Z}_{1\nu}\Delta N_\nu}{2\det Z}\right)^2 \quad (62)$$

$$2\Delta_\nu^{\pm}(\mathbf{\Delta N}, \mathbf{D}, \mathbf{N}^{\pm}) = 2N_\nu^{\pm} + \left(\mathcal{Z}_{1\nu}D_1 + \mathcal{Z}_{\nu\nu}D_\nu \pm \frac{\mathcal{Z}_{11}\Delta N_\nu - \mathcal{Z}_{\nu 1}\Delta N_1}{2\det Z}\right)^2 \quad (63)$$

The quantum numbers $\mathbf{\Delta N} = (\Delta N_1, \Delta N_\nu), \mathbf{D} = (D_1, D_\nu)$ and $\mathbf{N}^{\pm} = (N_1^{\pm}, N_\nu^{\pm})$ are the obvious generalization of the one defined in the previous section. For two coupled Gaussian models with a total central charge $c = 2$, the scaling dimensions for primary fields are given by

$$d(\mathbf{\Delta N}, \mathbf{D}) = (\mathcal{Z}_{11}D_1 + \mathcal{Z}_{\nu 1}D_\nu)^2 + \left(\frac{\mathcal{Z}_{\nu\nu}\Delta N_1 - \mathcal{Z}_{1\nu}\Delta N_\nu}{2\det Z}\right)^2 \quad (64)$$
$$(\mathcal{Z}_{1\nu}D_1 + \mathcal{Z}_{\nu\nu}D_\nu)^2 + \left(\frac{\mathcal{Z}_{11}\Delta N_\nu - \mathcal{Z}_{\nu 1}\Delta N_1}{2\det Z}\right)^2$$

When two sectors of the Gaussian model have different velocities v_1 and v_2, the correlation functions of primary fields are given by:

$$< \Phi_{\Delta^\pm}(x,t)\Phi_{\Delta^\pm}(0,0) >= \frac{\exp\left[-2i(\mathcal{D}_1 p_{F1} + \mathcal{D}_2 p_{F2})x\right]}{(x-iv_1t)^{2\Delta_1^+}(x+iv_1t)^{2\Delta_1^-}(x-iv_2t)^{2\Delta_2^+}(x+iv_2t)^{2\Delta_2^-}} \quad (65)$$

The vortices generate operators represented by exponents of the dual phases of the two gapless fields. We shall

call the scaling dimensions of this operators d_{Θ_1} and d_{Θ_ν} respectively. The conditions for irrelevance of vortices are :

$$d_{\Theta_1} > 2 \ , \ d_{\Theta_\nu} > 2 \tag{66}$$

To see the effect of the interaction between the order parameters on the stability of the superconducting state with respect to vortices we shall consider the dependence of the scaling dimension of the potentially relevant operators as a function of h_- for fixed $h_+ \gg M$. In particular we will compare the scaling dimensions in two extreme limits where Eqs (58-61) can be studied analytically. The limit of small h_-, which corresponds to the physical situation in which the two superconductors have the same bare critical temperature, and the limit $h_- > h_c$. Having fixed h_+ correspond to fix, for example, h_1 which is proportional to the mean field critical temperature of layer 1. We want to observe how the presence of the other superconductor affects the scaling dimension of the order parameter of this superconductor, i.e. his effective critical temperature.

For $h_- \ll 1$ the matrix problem given by Eqs. (58-61) can be reduced by to scalar one by Fourier transforming with respect to λ. In these circumstances the dressed charge matrix has the form:

$$Z = \begin{pmatrix} \zeta(B) & 0 \\ \zeta(B)/2 & \sqrt{\nu/2} \end{pmatrix} \tag{67}$$

where $\zeta(\theta)$ is determined by the following equation:

$$\zeta(\theta) = 1 + \int_{-B}^{B} \mathrm{d}\theta' R(\theta - \theta')\zeta(\theta') \tag{68}$$

and the kernel is defined by Eq.(55). In this limit the scaling dimension is given by:

$$d(\mathbf{\Delta N}, \mathbf{D}) = (\zeta(B)D_1 + \frac{\zeta(B)}{2}D_\nu)^2 + \frac{\nu}{2}D_\nu^2 \tag{69}$$

and, as shown in Appendix:

$$d_{\Theta_1} = d((0,0),(1,0)), \tag{70}$$
$$d_{\Theta_\nu} = d((0,0),(1,-2)). \tag{71}$$

Then in general the model will present two different BKT temperatures.

In the limit $B >> 1$ Eq.(68) can be solved analytically. Using again the WH method we obtain:

$$\zeta(B) = \sqrt{2\nu} \tag{72}$$

from which:

$$d_{\Theta_1} = d_{\Theta_2} = 2\nu = \frac{2}{1-g} \tag{73}$$

Then in this particular limit the conditions (66) are always satisfied and the system is always stable with respect to vortices. We notice that there are operators characterized by $D_1 = 1$ and $D_\nu = -1$ or $D_1 = 0$ and $D_\nu = 1$ with a smaller scaling dimension:

$$d((0,0),(1,-1)) = d((0,0),(0,1)) = \nu \equiv d_{12}^{\pm} \tag{74}$$

Again following the results in the Appendix one can easily see that these operators are associated with the bosonic exponents containing linear combinations of the dual phase of the two fields. These operators become relevant for $g < 1/2$.

For $h_- > h_c$ one again has only one gapless mode and repeating the procedure of the previous section we get, for $B \gg 1$:

$$d_{\Theta_1}(h > h_c) = \nu, \tag{75}$$

in agreement with the results obtained for the type I models.

Then the effect noticed at the mean field level survives a deeper analysis. The effective critical temperature of a superconductor is enhanced by the presence of another one. The effect is present only if the critical temperature of this superconductor is above some critical value and is maximal when the two superconductors have the same critical temperature.

V. DISCUSSION

We have studied some properties of two families of integrable models describing two dimensional superconductors interacting with an insulating substrate. The models show some interesting features that may be general properties of layered superconductors. Due to integrability of the models in question one can study them in a strong coupling regime. So most of the results presented here are non-perturbative, as suggested by the previous analysis [11].

The models presented here can be grouped into two families. The members of one family describe a single superconducting plane interacting with n insulating layers. This gives a possibility to study the dependence of the BKT transition temperature on n. Remarkably the presence of the insulating stratum makes the system more stable with respect to the BKT transition and the transition temperature increases with n. The models of the second family describe an insulator sandwiched between two superconductors, and from this it is possible to extract information about the effect of the interaction between the superconducting order parameters. We have considered only the simplest model of this family, given by $n = 0$. Even in this case the presence of the other superconductor stabilizes the system with respect to the BKT transition. For most of the results presented here it is essential that the systems we consider are two dimensional.

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APPENDIX

In this appendix we summarize some basic results on the correlation functions of exponents of bosonic fields in Gaussian field theories and present a simple way to identify the operators that correspond to various combinations of quantum numbers $\mathbf{\Delta N}$ and $\mathbf{D}$. Let us consider the Gaussian model:

$$S = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}x[v^{-1}(\partial_\tau\Phi)^2 + v(\partial_x\Phi)^2] \tag{76}$$

It is a well known that the correlation function of exponents of bosonic fields is given by:

$$\langle \exp[\mathrm{i}\beta_1\Phi(\xi_1)]...\exp[\mathrm{i}\beta_N\Phi(\xi_N)]\rangle = \tag{77}$$
$$\prod_{i>j}\left(\frac{z_{ij}\bar{z}_{ij}}{a^2}\right)^{(\beta_i\beta_j/4\pi)}\left(\frac{R}{a}\right)^{-(\sum_n \beta_n)^2/4\pi}$$

where $z = \tau + \mathrm{i}x/v$, $\bar{z} = \tau - \mathrm{i}x/v$ and in an infinite system this is different from zero only if:

$$\sum_n \beta_n = 0 \tag{78}$$

The expression (77) is factorized into analytic and anti-analytic parts and then it is useful to rewrite it as:

$$\begin{aligned}&\langle \exp[\mathrm{i}\beta_1\Phi(\xi_1)]...\exp[\mathrm{i}\beta_N\Phi(\xi_N)]\rangle \\ &= G(z_1,...,z_N)G(\bar{z}_1,...,\bar{z}_N)\delta_{\sum\beta_n,0}\end{aligned} \tag{79}$$

where

$$G(\{z\}) = \prod_{i>j}\left(\frac{z_{ij}}{a}\right)^{(\beta_i\beta_j/4\pi)}$$

This factorization guarantees that analytic and anti-analytic parts of the correlation functions can be studied independently. Since factorization of the correlation functions is a general fact, it can be formally written as factorization of the corresponding fields. Then inside the $\langle ... \rangle$-sign one can rewrite $\Phi(z,\bar{z})$ as a sum of independent analytic and anti-analytic fields:

$$\Phi(z,\bar{z}) = \phi(z) + \bar{\phi}(\bar{z}), \tag{80}$$
$$\exp[\mathrm{i}\beta\Phi(z,\bar{z})] = \exp[\mathrm{i}\beta\phi(z)]\exp[\mathrm{i}\beta\bar{\phi}(\bar{z})] \tag{81}$$

For many purposes it is convenient to introduce the 'dual' field $\Theta(z,\bar{z})$ defined as

$$\Theta(z,\bar{z}) = \phi(z) - \bar{\phi}(\bar{z}) \tag{82}$$

which satisfies the following equations:

$$\partial_\mu\Phi = -\mathrm{i}\epsilon_{\mu\nu}\partial_\nu\Theta \tag{83}$$

In order to study correlation functions of the analytic and the anti-analytic fields, we define the fields

$$\begin{aligned}A(\beta,z) &\equiv \exp\left\{\frac{\mathrm{i}}{2}\beta[\Phi(z,\bar{z}) + \Theta(z,\bar{z})]\right\} \\ \bar{A}(\bar{\beta},\bar{z}) &\equiv \exp\left\{\frac{\mathrm{i}}{2}\bar{\beta}[\Phi(z,\bar{z}) - \Theta(z,\bar{z})]\right\}\end{aligned} \tag{84}$$

with, generally speaking, different $\beta, \bar{\beta}$. With the operators $A(\beta,z), \bar{A}(\bar{\beta},\bar{z})$ one can expand local functionals of mutually nonlocal fields Φ and Θ. Suppose that $F(\Phi,\Theta)$ is a local functional periodic both in Φ and Θ with the periods T_1 and T_2, respectively. This functional can be expanded in terms of the bosonic exponents:

$$\begin{aligned}F(\Phi,\Theta) &= \sum_{n,m}\tilde{F}_{n,m}\exp[(2\mathrm{i}\pi n/T_1)\Phi + (2\mathrm{i}\pi m/T_2)\Theta] \\ &= \sum_{n,m}\tilde{F}_{n,m}A(\beta_{nm},z)\bar{A}(\bar{\beta}_{nm},\bar{z})\end{aligned} \tag{85}$$

where

$$\beta_{nm} = 2\pi\left(\frac{n}{T_1} + \frac{m}{T_2}\right) \tag{86}$$

$$\bar{\beta}_{nm} = 2\pi\left(\frac{n}{T_1} - \frac{m}{T_2}\right) \tag{87}$$

It turns out that the periods T_1, T_2 are not arbitrary, but related to each other. The reason for this lies in the fact that the correlation functions must be uniquely defined on the complex plane. We can see how this argument works using the pair correlation function as an example:

$$\begin{aligned}&\langle A(\beta_{nm},z_1)\bar{A}(\bar{\beta}_{nm},\bar{z}_1)A(-\beta_{nm},z_2)\bar{A}(-\bar{\beta}_{nm},\bar{z}_2)\rangle \\ &= (z_{12})^{-\beta_{nm}^2/4\pi}(\bar{z}_{12})^{-\bar{\beta}_{nm}^2/4\pi} = \frac{1}{|z_{12}|^{2d}}\left(\frac{z_{12}}{\bar{z}_{12}}\right)^S\end{aligned} \tag{88}$$

where we introduce the quantities

$$d = \Delta^+ + \Delta^- = \frac{1}{8\pi}(\beta^2 + \bar{\beta}^2)$$

and

$$S = \Delta^+ - \Delta^- = \frac{1}{8\pi}(\beta^2 - \bar{\beta}^2)$$

which are called the 'scaling dimension' and the 'conformal spin', respectively.

The two branch cut singularities in Eq. (88) cancel each other and give a uniquely defined function only if

$$2S = (\text{integer}) \tag{89}$$

i.e., physical fields with uniquely defined correlation functions must have integer or half-integer conformal spins. This equation suggests the relation

$$T_1 = \frac{4\pi}{T_2} \equiv \sqrt{4\pi K} \tag{90}$$

as the minimal solution. Here we introduce the Luttinger liquid parameter K for future convenience. The normalization is such that at $K = 1$ the periods for the field Φ and its dual are equal. The quantities Δ^+, Δ^- are called 'conformal dimensions' or 'conformal weights'. In the case of the Gaussian model (76) the conformal dimensions of the basic operators are given by:

$$\Delta^+_{nm} \equiv \beta^2_{nm}/8\pi = \frac{1}{8}\left(m\sqrt{K} + \frac{n}{\sqrt{K}}\right)^2$$
$$\Delta^-_{nm} \equiv \bar{\beta}^2_{nm}/8\pi = \frac{1}{8}\left(m\sqrt{K} - \frac{n}{\sqrt{K}}\right)^2 \tag{91}$$

This equations can be rewritten in terms of the quantum numbers ΔN and D previously introduced as:

$$2\Delta^{\pm}_{nm} = \left(\frac{D\sqrt{K}}{2} \pm \frac{\Delta N}{\sqrt{K}}\right)^2 \tag{92}$$

In integrable models the parameter K is related to the dressed charge introduced in Sec.III:

$$K = 4\zeta(B)^2 \tag{93}$$

from which we obtain the form (92):

$$2\Delta^{\pm} = \left(\zeta(B)D \pm \frac{\Delta N}{2\zeta(B)}\right)^2. \tag{94}$$

Comparing it with equation (85) we can easily see that the scaling dimension d_Φ of the field Φ is given by $\Delta N = 1$ and $D = 0$, while the one of the dual field, d_Θ, by $\Delta N = 0$ and $D = 1$:

$$d_\Phi = 1/4\zeta(B)^2 \tag{95}$$
$$d_\Theta = \zeta(B)^2. \tag{96}$$

All the considerations above can be generalized to the case of n gaussian fields with the same velocity:

$$S = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}x \sum_{i=1}^{n}[v^{-1}(\partial_\tau\Phi_n)^2 + v(\partial_x\Phi_n)^2] \tag{97}$$

In a similar fashion as before we can introduce the fields:

$$A(\mathbf{B}, z) \equiv \exp\left\{\frac{\mathrm{i}}{2}\mathbf{B}\cdot[\mathbf{\Phi}(z,\bar{z}) + \mathbf{\Theta}(z,\bar{z})]\right\} \tag{98}$$
$$\bar{A}(\bar{\mathbf{B}}, \bar{z}) \equiv \exp\left\{\frac{\mathrm{i}}{2}\bar{\mathbf{B}}\cdot[\mathbf{\Phi}(z,\bar{z}) - \mathbf{\Theta}(z,\bar{z})]\right\}$$

where $\mathbf{\Phi} = (\Phi_1, ..., \Phi_n)$, $\mathbf{\Theta} = \Theta_1, ..., \Theta_n$, $\mathbf{B} = (\beta_1, ..., \beta_n)$ and $\bar{\mathbf{B}} = (\bar{\beta}_1, ..., \bar{\beta}_n)$. The generalization of the conformal dimension and spin are:

$$d(\mathbf{\Delta N}, \mathbf{D}) = 1/4\mathbf{\Delta N}^T\mathbf{X}^{-1}\mathbf{\Delta N} + \mathbf{D}^T\mathbf{X}\mathbf{D} \equiv \mathbf{\Delta}^+ + \mathbf{\Delta}^-$$
$$S(\mathbf{\Delta N}, \mathbf{D}) = \mathbf{\Delta N}^T\mathbf{D}$$

where $\mathbf{\Delta N} = (\Delta N_1, ..., \Delta N_n)^T$, $\mathbf{D} = (D_1, ..., D_n)^T$ and $\mathbf{X} = Z^T Z$ with Z is the dressed charge matrix defined by (57). Then the conformal dimension of each field is given by:

$$\Delta^+_i = \frac{1}{8\pi}\beta_i^2\ ;\ \Delta^-_i = \frac{1}{8\pi}\bar{\beta}_i^2 \tag{99}$$

For the case of two gapless modes of interest in the paper the forms (98) and (99) reduces to

$$A(\mathbf{B}, z)\bar{A}(\bar{\mathbf{B}}, \bar{z}) = \tag{100}$$
$$\exp\left[i(Z_{11}D_1 + Z_{21}D_2)\Theta_1 + i(Z_{22}\Delta N_1 - Z_{12}\Delta N_2)\Phi_1 + i(Z_{12}D_1 + Z_{21}d_2)\Theta_2 + i(Z_{11}\Delta N_2 - Z_{21}\Delta N_1)\Phi_2\right]$$

It is easy to read from (100) that the exponent of the dual field Θ_1 is characterized by $Z_{12}D_1 + Z_{21}D_2 = 0$ and $\Delta N_{1,2} = 0$, while for Θ_2 you need $Z_{11}D_1 + Z_{21}D_2 = 0$ and $\Delta N_{1,2} = 0$. Then for a dressed charge matrix of the form (67) the conformal dimensions for exponents of the two dual fields are

$$d_{\Theta_1} = d((0,0),(1,0)) \tag{101}$$
$$d_{\Theta_2} = d((0,0),(-1,2)). \tag{102}$$

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